A Paradox of Counting: Voting Methods and Fair Decisions

Robert T. Schroder

Boy Scout Troop 577

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For the purpose of this experiment, the following scenario was presented: *Your unit wants to plan a superactivity for next summer but cannot agree on what that activity should be. There are four options under consideration, and your unit decides to vote. Your task is to collect ballots and tabulate results using several different voting methods* (Boy Scouts of America, 2019).

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Creating the Survey

A two-question survey was created with the intent to capture the following data from particpatnts:

1. Their role in the unit (Scout, Parent, Unit Leader)
2. First through Fourth choice for the type of summer activity they would prefer.

Review of Survey[1]

The survey was reviewed by the Scoutmaster & Committee Chair prior to sending out to the unit. Positive feedback was received and approval to send to the unit via email, slack, and SMS.

Distribution

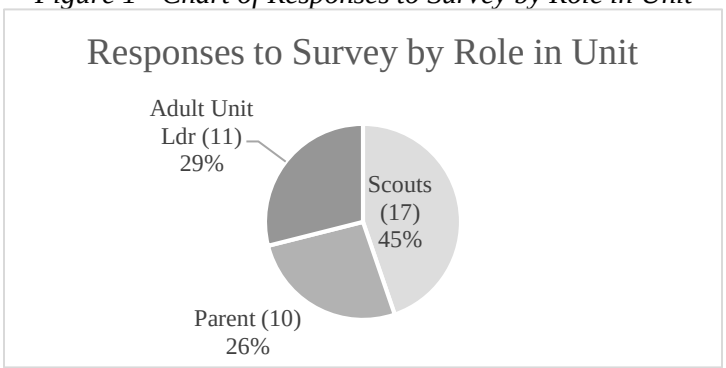
The survey was sent to roughly 89 scouts and their parents on the troop email list, as well as texted to 30 scouts via the new Troopmaster web interface. The survey was also posted in the unit slack workspace in the announcement channel.

Results of Survey

Survey Respondents

Of those who received the survey, 39 responses were received. A breakdown is shown in the following figure:

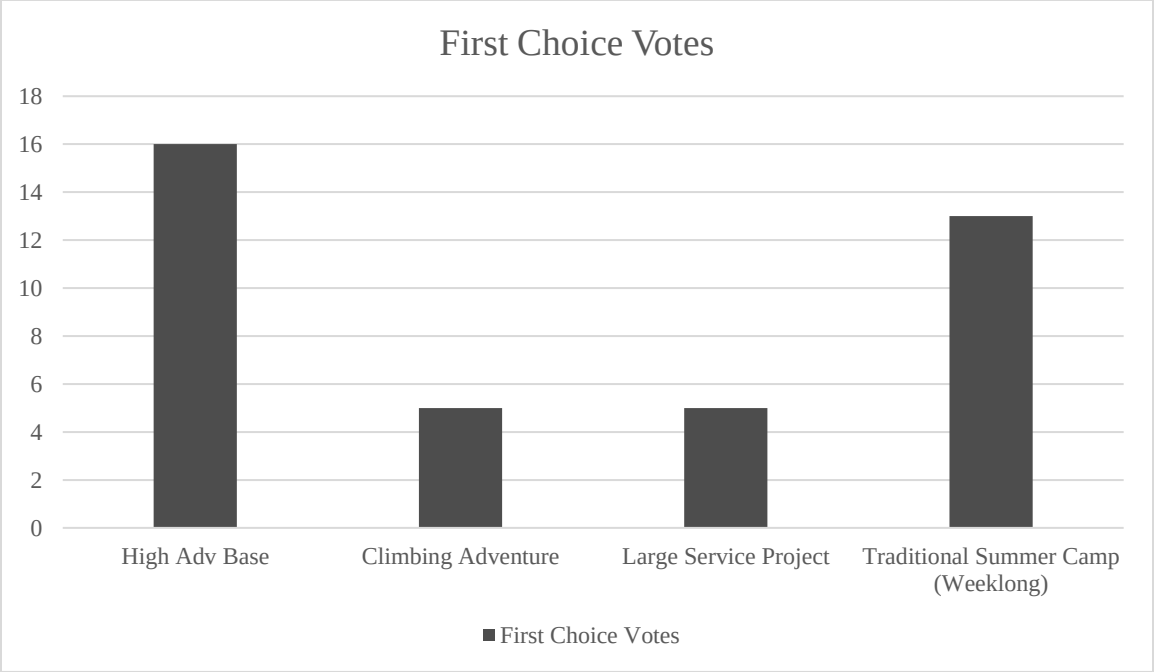
Figure 1 - Chart of Responses to Survey by Role in Unit



Plurality Method

The plurality method is defined as "For each ballot, only the first place vote counted. The "candidate" with the most first place votes is the winner. If ballots are combined into a preference schedule then the Plurality method will ignore all non-1st place votes" (Csima, 2019). The following first-choice votes were collected in the survey:

Figure 2 - First Choice Votes for each category



Looking at the responses, there is a plurality in favor of the High Adventure Base. Some bias may be present in this due to data collection. This survey was sent to the unit by email, which more senior scouts are likely to check compared to younger scouts who may want to do something different.

Borda Method

A preference table of the votes looked as follows:

Figure 3 - Preference Table of Results

Position	Points	4	4	4	2	2	1	2	2	1	1	1	1	4	2	3	2	3
1st	4	A	A	A	A	A	B	B	D	D	B	B	C	D	C	D	C	D
2nd	3	B	B	D	C	D	A	A	A	A	C	D	B	B	D	C	D	C
3rd	2	C	D	B	D	C	C	D	B	C	A	A	A	A	A	A	B	B
4th	1	D	C	C	B	B	D	C	C	B	D	C	D	C	B	B	A	A

This table will be used for the borda count and later methods. The borda method is performed by assigning the first place vote a score of *N*, the second *N-1*, and so on until reaching a score of 1 for the last choice. For example, in column 1 of results, the ballot A-B-C-D was submitted 4 times. This results in 16 points for A (4pts * 4 times), 12 points for B (3pts * 4), etc.

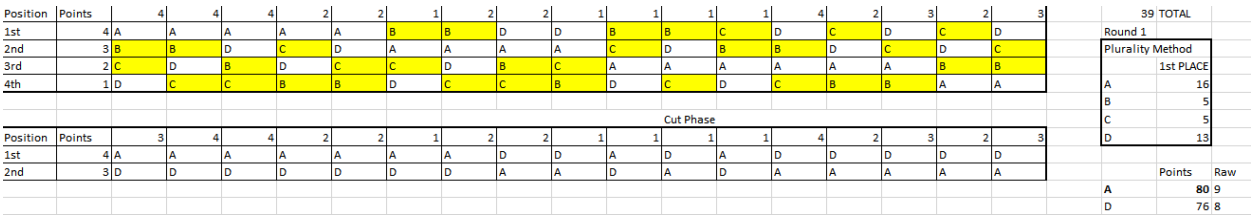
The results of a borda count returns the following for the given preference table:

Figure 4 - Borda Count Table

Borda Count Method	
Total Points	
A	111
B	91
C	80
D	108

A was the winner, the High Adventure Base. Option D, the traditional summer camp option lagged by only one second place vote to force a tie.

Plurality with Elimination



Position	Points																		
1st	4	A	A	A	A	A	B	B	D	D	B	B	C	D	C	D	C	D	
2nd	3	B	B	D	C	D	A	A	A	A	C	D	B	B	D	C	D	C	
3rd	2	C	D	B	D	C	C	D	B	C	A	A	A	A	A	A	B	B	
4th	1	D	C	C	B	B	D	C	C	B	D	C	D	C	B	B	A	A	

Cut Phase

Position	Points																		
1st	4	A	A	A	A	A	A	A	D	D	A	D	A	D	D	D	D	D	
2nd	3	D	D	D	D	D	D	D	A	A	D	A	D	A	A	A	A	A	

	39	TOTAL
Round 1		
Plurality Method		
		1st PLACE
A		16
B		5
C		5
D		13

	Points	Raw
A	**80**	9
D	76	8

PwE required the removal of all lower options, and shifting the remaining votes "up" on the preference table. This resulted in the elimination of B & C, as these choices tied for 3rd/4th place.

Option A was still the winner, albeit by only one vote (raw or borda method).

Pairwise

A pairwise table compares options as a head-to-head comparsion. Each letter is compared from the row to the header (i.e. B v A = 0).

Figure 5 - Pairwise Matchup Result

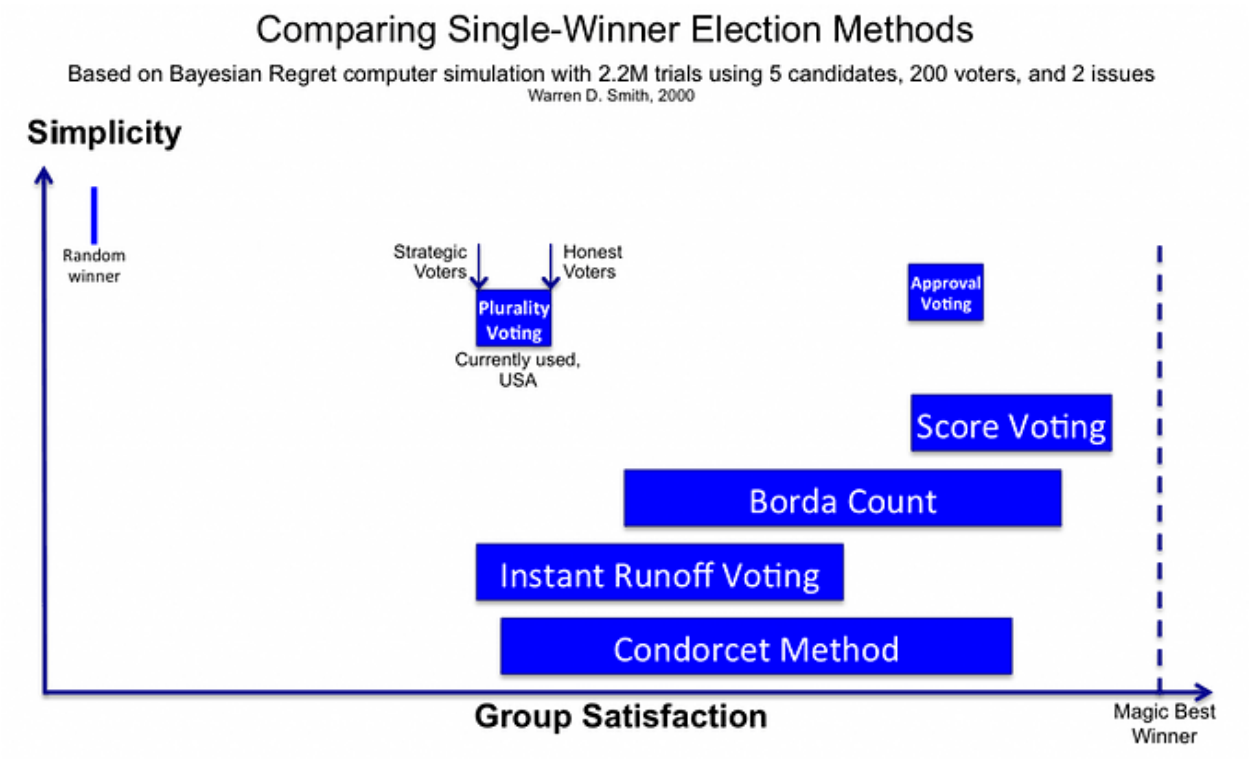
	A	B	C	D		
A	-	1	1	1	A	3
B	0	-	0.5	0	B	0.5
C	0	0.5	-	0	C	0.5
D	0	1	1	-	D	2

Analysis of Results

What do you notice? How fair is each method?

A significant point of interest is the split result between a traditional summer camp (D) and a high adventure base (A). A hypothesis for this is that the more older/senior scouts voted for a high adventure base (as they are more likely to check their email or text or slack notifications and fill out a form, compared to their younger peers); whereas parents of younger scouts most likely made the traditional summer camp choice on their behalf. Our unit has expressed interest in a climbing trip before, but not much else has been discussed at this time, most likely leading to a lower score on the survey compared to other traditional options.

Each method resulted in the plurality winner, A/High Adventure, being the winning choice. Personally, the Borda method seems fairer when compared to simple majority. A study done by Dr. Warren Smith of Princeton seemed incline to agree:



As discussed in the next question, these methods are most effective with honest voters versus strategic voters.

How would the results be affected if two or three voters had cast strategic ballots (instead of sincere ballots), in an effort to "not waste their votes"?

Most election scientists define strategic ballots as "when voters cast an insincere ballot in order to increase their expected value for an election outcome. A common example of this behavior is when supporters of a minor party candidate vote for their favorite major party candidate, based on the impression that the minor party candidate is unlikely to win" (The Center for Election Science, 2019).

The results would have been skewed if either of the climbing or service project group members voted strategically instead of sincerely. Either group could have secured a win for the larger majority groups.

Which of the four voting methods do you believe is the right voting method for this decision in your unit? Why?

While the Borda method does take more time to compute a winner, it would most likely end up with a better method of selecting an activity in the unit.

Consider how we elect the president of the United States of America.

What voting method do we use?

In the US, we use an indirect method of voting. Ballots are cast for the electors chosen to vote in the U.S. Electoral College. Each state has it's own method of choosing electors, and D.C. is represented with three votes.

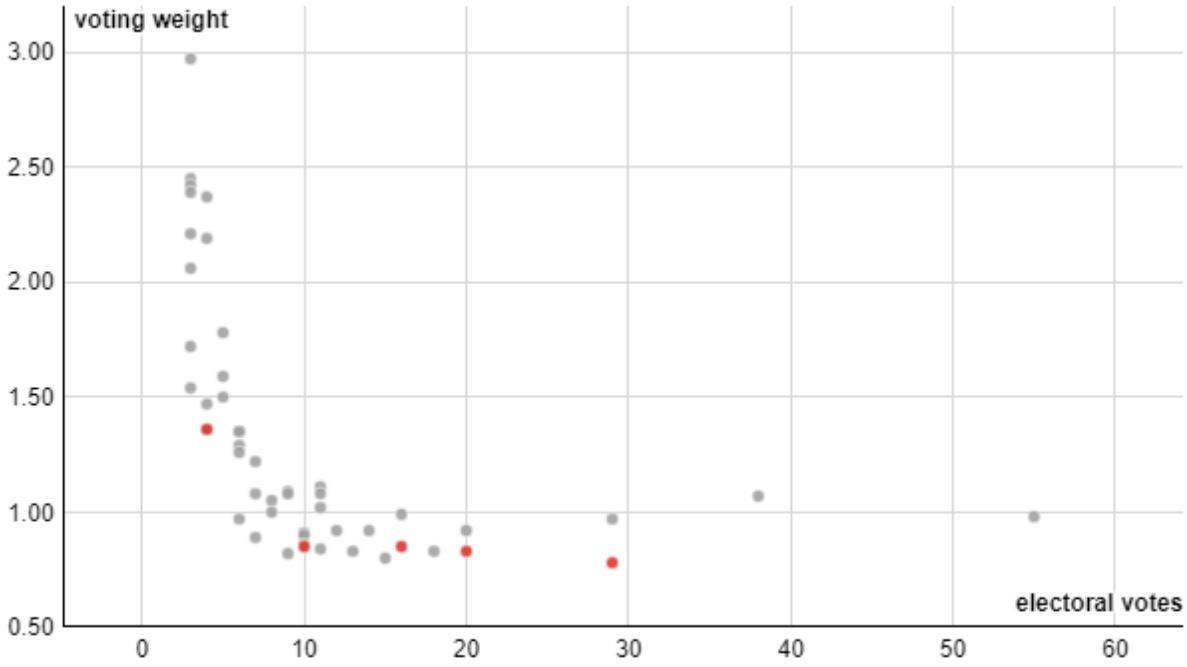
What are its advantages and disadvantages?

The primary advantage of the electoral college is ensuring that smaller states in the middle US are represented in the presidential process.

A primary disadvantage is that it is not a true democracy, as the popular vote and electoral vote can (and have been) two different results.

Do you believe each voting citizen in the United States has an equal say in the vote tabulation?

Citizens have an inequal say based on their location (Electoral votes). This graph shows a negative power regression on the states. Interestingly, midsize states such as Florida fare the worst.



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Source: Author's analysis uses data from United States Elections Project Get the data

Is it possible for citizens to cast strategic votes and influence the outcome of a presidential election?

Yes, it is possible. With write-in/third-party candidates, it is theoretically possible to strategic-vote and influence. However, the two party system currently does not allow much room to do so, much less on a large/coordinated scale.

References

Boy Scouts of America. (2019). *Scouts BSA Supernova Activity Topics*. Retrieved from Boy Scouts of America: https://www.scouting.org/stem-nova-awards/awards/scouts-bsa-supernova-topics/

Coconino Community College. (n.d.). Math 142. In C. C. College. Retrieved from https://www.coconino.edu/resources/files/pdfs/academics/arts-and-sciences/MAT142/Chapter_7_VotingSystems.pdf

Csima, E. (. (2019). *Math 111 Voting Lecture 2*. Retrieved from University of Kentucky, Department of Mathematics: http://ms.uky.edu/~csima/ma111/VotingLecture2.pdf

Figures title: